(article begins on next page)

An Iterative Convex Approach for Fixed-Order Robust $\mathscr{H}_2/\mathscr{H}_\infty$ Control of Discrete-Time Linear Systems with Parametric Uncertainty

Gijs Hilhorst*, Goele Pipeleers, Wim Michiels, Ricardo C. L. F. Oliveira, Pedro L. D. Peres and Jan Swevers

***Abstract*— This paper presents a convex approach to design fixed-order robust $\mathscr{H}_2/\mathscr{H}_\infty$ controllers for discrete-time linear time-invariant (LTI) systems affected by parametric uncertainty. Starting from an a priori computed stabilizing full-order parameter-dependent controller for the same system, which is designed under the assumption that the parameter is exactly known, parameter-dependent sufficient linear matrix inequalities (LMIs) for robust $\mathscr{H}_2/\mathscr{H}_\infty$ analysis and synthesis are presented. A novel LMI procedure is proposed to iteratively compute less conservative robust controllers, utilizing a feasible solution of the fixed-order synthesis conditions as a starting point. Assuming polynomial parameter dependencies of all system matrices, tractable LMI formulations that guarantee feasibility of the parameter-dependent conditions are derived using well-known relaxations based on Pólya's theorem. Numerical comparisons with existing methods confirm the potential of the proposed robust controller design approach.**

I. INTRODUCTION

The general fixed-order controller design problem for LTI systems is challenging, even in the case of a single performance objective and without parametric uncertainty. This problem has already attracted many researchers since decades, and still constitutes a field of active research due to its complexity.

Specifically, for accurately known LTI systems, the existence of a convex reformulation of the fixed-order controller design problem is unknown. Despite the lack of such a convex condition, various approaches have been developed, which include solving the nonconvex problem directly [1], [2], [3], or deriving convex sufficient conditions [4], [5]. Although the latter (conservative) convex approaches proved successful for accurately known LTI systems, their extension to cope with parametric uncertainty is not evident [6].

Recently, an iterative LMI approach for robust static output feedback design is presented for continuous-time LTI systems subject to parametric uncertainty in [7], relying on an a priori computed parameter-dependent state feedback controller for a specific augmented system. This approach is readily extendable to design fixed-order robust controllers, and is based on the idea proposed in [4] for accurately known LTI systems. Additionally, sufficient LMIs for robust static output feedback design are proposed in [8], [9], incorporating scalar parameters in the LMIs to reduce conservatism at the expense of a higher numerical burden. In [10], an iterative LMI approach is presented to gradually compute fixed-order robust controllers with better performance for discrete-time polytopic systems. However, the latter approach does not allow all system matrices to be parameter-dependent.

In this paper, the LMI framework presented in [5], [11], [12] is extended with an iterative LMI procedure to allow the design of high performance robust $\mathscr{H}_2/\mathscr{H}_\infty$ controllers for LTI systems with parametric uncertainty. In contrast to the aforementioned approaches, the resulting design approach handles any prefixed controller order, allows polynomial parameter dependencies of all system matrices, considers multiple performance objectives, and allows reductions of conservatism by iterative computation of robust controllers with better performance. The benefits of our approach are illustrated by numerical comparisons with existing methods.

The paper is organized as follows. First, Section II discusses the mathematical problem formulation. Then, the fixed-order robust controller design approach is presented in Section III, followed by numerical validations in Section IV. The conclusions are given in Section V.

Notation

The set of nonnegative (positive) integers is denoted by $\mathbb{N}$ ($\mathbb{N}_+$), while $\mathbb{R}^n$ ($\mathbb{R}^{m\times n}$) is the set of real vectors (matrices) of dimension n ($m\times n$). I_n denotes the identity matrix of dimension $n\times n$ and $0_{m\times n}$ denotes a zero matrix of dimension $m\times n$. The subscripts are omitted when the dimensions can be inferred from the context. The transpose of a matrix X is written as X', and the notation $\mathrm{He}\{X\} = X + X'$ is used to shorten formulas. The sets of real symmetric (real positive definite) matrices of dimension n are denoted by $\mathbb{S}^n$ ($\mathbb{S}^n_+$). A star ($\star$) indicates symmetric terms in matrix inequalities.

II. PROBLEM FORMULATION

We consider the finite-dimensional uncertain discrete-time linear system

$$\begin{cases} x(k+1) = A(\alpha)x(k) + B_w(\alpha)w(k) + B_u(\alpha)u(k), \\ \quad z(k) = C_z(\alpha)x(k) + D_w(\alpha)w(k) + D_u(\alpha)u(k), \\ \quad y(k) = C_y(\alpha)x(k) + D_y(\alpha)w(k), \end{cases} \tag{1}$$

*This work benefits from research project G0.0712.11 of the Research Foundation-Flanders (FWO - Vlaanderen); KU Leuven-BOF PFV/10/002 Center-of-Excellence Optimization in Engineering (OPTEC); the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office (DYSCO); IWT SBO project MBSE4Mechatronics: Model-based Systems Engineering for Mechatronics; and the Brazilian agencies CNPq and CAPES. G. Pipeleers is a Postdoctoral Fellow of the Research Foundation-Flanders (FWO - Vlaanderen).

G. Hilhorst, G. Pipeleers and J. Swevers are with the Department of Mechanical Engineering, KU Leuven, 3001 Heverlee, Belgium

[2]W. Michiels is with the Department of Computer Science, KU Leuven, Belgium.

[3]R. C. L. F. Oliveira and P. L. D. Peres are with the School of Electrical and Computer Engineering, University of Campinas – UNICAMP, 13083-852, Campinas, SP, Brazil.

*Corresponding author: gijs.hilhorst@kuleuven.be

$k \in \mathbb{N}$, with state $x(k) \in \mathbb{R}^{n_x}$, exogenous input $w(k) \in \mathbb{R}^{n_w}$, control input $u(k) \in \mathbb{R}^{n_u}$, regulated output $z(k) \in \mathbb{R}^{n_z}$ and measured output $y(k) \in \mathbb{R}^{n_y}$. All system matrices are assumed to have a polynomial dependency on the time-invariant parameter $\alpha \in \Omega$, where $\Omega \subset \mathbb{R}^N$ is a bounded convex polytope.

The aim is to design robust dynamic output feedback controllers

$$K : \begin{cases} x_{\mathrm{c}}(k+1) = A_{\mathrm{c}} x_{\mathrm{c}}(k) + B_{\mathrm{c}} y(k), \\ u(k) = C_{\mathrm{c}} x_{\mathrm{c}}(k) + D_{\mathrm{c}} y(k), \end{cases} \tag{2}$$

with a preselected fixed order q $(0 \leq q \leq n_x)$ that stabilize the uncertain system (1) and satisfy one or more closed-loop $\mathscr{H}_2$ and/or $\mathscr{H}_\infty$ performance specifications for all $\alpha \in \Omega$. Hence, we consider optimization of the worst-case performance.

Grouping the controller matrices of (2) as

$$\Theta := \begin{bmatrix} A_{\mathrm{c}} & B_{\mathrm{c}} \\ C_{\mathrm{c}} & D_{\mathrm{c}} \end{bmatrix}, \tag{3}$$

the closed-loop interconnection of the uncertain system (1) with the controller (2) is written as

$$H_\Theta(\alpha) : \begin{cases} x_{\mathrm{cl}}(k+1) = \mathscr{A}_\Theta(\alpha) x_{\mathrm{cl}}(k) + \mathscr{B}_\Theta(\alpha) w(k), \\ z(k) = \mathscr{C}_\Theta(\alpha) x_{\mathrm{cl}}(k) + \mathscr{D}_\Theta(\alpha) w(k), \end{cases} \tag{4}$$

where $x_{\mathrm{cl}}(k) = \begin{bmatrix} x(k)' & x_{\mathrm{c}}(k)' \end{bmatrix}' \in \mathbb{R}^{n_x+q}$ is a closed-loop state vector. Defining the matrices

$$\begin{bmatrix} \tilde{A}(\alpha) & \tilde{B}_w(\alpha) & \tilde{B}_u(\alpha) \\ \tilde{C}_z(\alpha) & \tilde{D}_w(\alpha) & \tilde{D}_u(\alpha) \\ \tilde{C}_y(\alpha) & \tilde{D}_y(\alpha) & 0 \end{bmatrix} := \left[\begin{array}{cc|c|cc} A(\alpha) & 0 & B_w(\alpha) & 0 & B_u(\alpha) \\ 0 & 0 & 0 & I_q & 0 \\ \hline C_z(\alpha) & 0 & D_w(\alpha) & 0 & D_u(\alpha) \\ \hline 0 & I_q & 0 & 0 & 0 \\ C_y(\alpha) & 0 & D_y(\alpha) & 0 & 0 \end{array}\right], \tag{5}$$

the affine dependency of the closed-loop matrices in (4) on Θ is expressed as

$$\begin{bmatrix} \mathscr{A}_\Theta(\alpha) & \mathscr{B}_\Theta(\alpha) \\ \mathscr{C}_\Theta(\alpha) & \mathscr{D}_\Theta(\alpha) \end{bmatrix} = \begin{bmatrix} \tilde{A}(\alpha) & \tilde{B}_w(\alpha) \\ \tilde{C}_z(\alpha) & \tilde{D}_w(\alpha) \end{bmatrix} + \begin{bmatrix} \tilde{B}_u(\alpha) \\ \tilde{D}_u(\alpha) \end{bmatrix} \Theta \begin{bmatrix} \tilde{C}_y(\alpha) & \tilde{D}_y(\alpha) \end{bmatrix}. \tag{6}$$

For any fixed value of $\alpha \in \Omega$, the $\mathscr{H}_2$ and $\mathscr{H}_\infty$ norm of $H_\Theta(\alpha)$ are indicated by $\|H_\Theta(\alpha)\|_2$, respectively, $\|H_\Theta(\alpha)\|_\infty$. The corresponding worst-case $\mathscr{H}_2$ and $\mathscr{H}_\infty$ performances of $H_\Theta(\alpha)$ are respectively given by $\max_{\alpha\in\Omega} \|H_\Theta(\alpha)\|_2$ and $\max_{\alpha\in\Omega} \|H_\Theta(\alpha)\|_\infty$.

III. FIXED-ORDER ROBUST $\mathscr{H}_2/\mathscr{H}_\infty$ CONTROL

In this section, we present a convex approach to design fixed-order robust $\mathscr{H}_2/\mathscr{H}_\infty$ controllers of the form (2) for the uncertain discrete-time system (1), guaranteeing exponential stability as well as multiple worst-case $\mathscr{H}_2$ and/or $\mathscr{H}_\infty$ performances of the closed-loop system. The proposed approach relies on a stabilizing parameter-dependent full-order controller for the same system, which is computed using, e.g., the convex approaches discussed in [13], [14].

A. Robust $\mathscr{H}_2$ and $\mathscr{H}_\infty$ analysis

This subsection presents extended parameter-dependent LMIs for worst-case $\mathscr{H}_2$ and $\mathscr{H}_\infty$ performance analysis of the uncertain linear system (1) in closed loop with a robust controller Θ, defined in (3). The proposed parameter-dependent LMIs are derived by linking the closed-loop performance of Θ to a (possibly unstable/destabilizing) parameter-dependent full-order controller for the same system (1), which is designed (e.g., see [13], [14]) under the assumption that the value of $\alpha \in \Omega$ is known. This parameter-dependent controller is allowed to have a polynomial dependency on $\alpha \in \Omega$, and is represented by a parameter-dependent matrix $\Psi(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_y)}$, defined similar as in (3). The closed-loop interconnection of (1) with $\Psi(\alpha)$ is denoted by $H_\Psi(\alpha)$. To characterize stability and performance of $H_\Theta(\alpha)$ in terms of $H_\Psi(\alpha)$, we augment Θ with Schur stable unobservable and/or uncontrollable dynamics to form a so-called lifted parameter-dependent controller matrix $\Theta_{\mathrm{a}}(\alpha)$ with the same dimensions as $\Psi(\alpha)$:

$$\Theta_{\mathrm{a}}(\alpha) := \left[\begin{array}{cc|c} A_c & A_{12}(\alpha) & B_c \\ 0 & A_{22}(\alpha) & 0 \\ \hline C_c & C_2(\alpha) & D_c \end{array}\right], \tag{7}$$

where $A_{22}(\alpha)$ is Schur stable for all $\alpha \in \Omega$. Note that $H_\Theta(\alpha)$ and $H_{\Theta_{\mathrm{a}}}(\alpha)$ share the same stability and performance properties. Defining $\Upsilon(\alpha) := \Theta_{\mathrm{a}}(\alpha) - \Psi(\alpha)$, it follows from (6) that

$$\begin{bmatrix} \mathscr{A}_{\Theta_{\mathrm{a}}}(\alpha) & \mathscr{B}_{\Theta_{\mathrm{a}}}(\alpha) \\ \mathscr{C}_{\Theta_{\mathrm{a}}}(\alpha) & \mathscr{D}_{\Theta_{\mathrm{a}}}(\alpha) \end{bmatrix} = \begin{bmatrix} \mathscr{A}_\Psi(\alpha) & \mathscr{B}_\Psi(\alpha) \\ \mathscr{C}_\Psi(\alpha) & \mathscr{D}_\Psi(\alpha) \end{bmatrix} + \begin{bmatrix} \tilde{B}_u(\alpha) \\ \tilde{D}_u(\alpha) \end{bmatrix} \Upsilon(\alpha) \begin{bmatrix} \tilde{C}_y(\alpha) & \tilde{D}_y(\alpha) \end{bmatrix}. \tag{8}$$

Based on relation (8), extended parameter-dependent LMI characterizations that guarantee an upper bound on the worst-case $\mathscr{H}_2$ and $\mathscr{H}_\infty$ performance of the discrete-time uncertain linear system (1) in closed-loop with a given robust controller (2) are presented in the following theorems. The presentation of these characterizations is facilitated by defining the following parameter-dependent matrices

$$Q_2(\alpha, \Psi(\alpha)) := \begin{bmatrix} I & 0 & 0 \\ \mathscr{A}_\Psi(\alpha) & \mathscr{B}_\Psi(\alpha) & \tilde{B}_u(\alpha) \\ 0 & I & 0 \end{bmatrix},$$

$$Q_\infty(\alpha, \Psi(\alpha)) := \begin{bmatrix} I & 0 & 0 \\ \mathscr{A}_\Psi(\alpha) & \mathscr{B}_\Psi(\alpha) & \tilde{B}_u(\alpha) \\ 0 & I & 0 \\ \mathscr{C}_\Psi(\alpha) & \mathscr{D}_\Psi(\alpha) & \tilde{D}_u(\alpha) \end{bmatrix}.$$

Theorem 1 (Extended robust $\mathscr{H}_2$ analysis conditions): Let $\Psi(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_y)}$ be an arbitrary parameter-dependent matrix, and let $\Theta_{\mathrm{a}}(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_y)}$ be constructed from $\Theta \in \mathbb{R}^{(q+n_u)\times(q+n_y)}$ by adding Schur stable uncontrollable and/or unobservable dynamics. Then, the closed-loop system $H_\Theta(\alpha)$, defined as in (4), is exponentially stable and $\|H_\Theta(\alpha)\|_2^2 < \mu$ for all $\alpha \in \Omega$ if there exist parameter-dependent matrices $P(\alpha) \in \mathbb{S}_+^{2n_x}$, $W(\alpha) \in \mathbb{S}^{n_z}$, $X_1(\alpha) \in \mathbb{R}^{2n_x\times(n_x+n_u)}$, $X_2(\alpha) \in \mathbb{R}^{n_w\times(n_x+n_u)}$, $X_3(\alpha) \in$

$$Q_2(\alpha,\Psi(\alpha))' \begin{bmatrix} -P(\alpha) & 0 & 0 \\ 0 & P(\alpha) & 0 \\ 0 & 0 & -I \end{bmatrix} Q_2(\alpha,\Psi(\alpha)) + \mathrm{He}\left\{ \begin{bmatrix} X_1(\alpha) \\ X_2(\alpha) \\ X_3(\alpha) \end{bmatrix} \begin{bmatrix} \Upsilon(\alpha)\tilde{C}_y(\alpha) & \Upsilon(\alpha)\tilde{D}_y(\alpha) & -I \end{bmatrix} \right\} \prec 0 \tag{9a}$$

$$\begin{bmatrix} W(\alpha) & \mathscr{C}_\Psi(\alpha) & \mathscr{D}_\Psi(\alpha) & \tilde{D}_u(\alpha) \\ \star & P(\alpha) & 0 & 0 \\ \star & \star & I & 0 \\ \star & \star & \star & 0 \end{bmatrix} + \mathrm{He}\left\{ \begin{bmatrix} X_4(\alpha) \\ X_5(\alpha) \\ X_6(\alpha) \\ X_7(\alpha) \end{bmatrix} \begin{bmatrix} 0 & \Upsilon(\alpha)\tilde{C}_y(\alpha) & \Upsilon(\alpha)\tilde{D}_y(\alpha) & -I \end{bmatrix} \right\} \succ 0 \tag{9b}$$

$\mathbb{R}^{(n_x+n_u)\times(n_x+n_u)}$, $X_4(\alpha) \in \mathbb{R}^{n_z\times(n_x+n_u)}$, $X_5(\alpha) \in \mathbb{R}^{2n_x\times(n_x+n_u)}$, $X_6(\alpha) \in \mathbb{R}^{n_w\times(n_x+n_u)}$ and $X_7(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_u)}$, for all $\alpha \in \Omega$, such that $\mathrm{Trace}\{W(\alpha)\} < \mu$ and the parameter-dependent LMIs (9a) and (9b) hold for all $\alpha \in \Omega$.

Theorem 2 (Extended robust $\mathscr{H}_\infty$ analysis conditions): Let $\Psi(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_y)}$ be an arbitrary parameter-dependent matrix, and let $\Theta_{\mathrm{a}}(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_y)}$ be constructed from $\Theta \in \mathbb{R}^{(q+n_u)\times(q+n_y)}$ by adding Schur stable uncontrollable and/or unobservable dynamics. Then, the closed-loop system $H_\Theta(\alpha)$, defined as in (4), is exponentially stable and $\|H_\Theta(\alpha)\|_\infty^2 < \gamma$ for all $\alpha \in \Omega$ if there exist matrices $P(\alpha) \in \mathbb{S}_+^{2n_x}$, $X_1(\alpha) \in \mathbb{R}^{2n_x\times(n_x+n_u)}$, $X_2(\alpha) \in \mathbb{R}^{n_w\times(n_x+n_u)}$ and $X_3(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_u)}$, for all $\alpha \in \Omega$, such that the parameter-dependent LMI (10) on page 4 holds for all $\alpha \in \Omega$.

The proofs of Theorem 1 and Theorem 2 are omitted, since they proceed along the same lines as the proofs presented in [5] for LTI systems. To provide some insight, note that eliminating the slack variables $X_j(\alpha)$ from the LMIs (9) (LMIs (10)) by application of the projection lemma [15] yields well-known equivalent $\mathscr{H}_2$ ($\mathscr{H}_\infty$) analysis conditions for $H_{\Theta_{\mathrm{a}}}(\alpha)$, and hence for $H_\Theta(\alpha)$, relying on a parameter-dependent Lyapunov function (e.g., see [16], [17] and references therein). While the choice of the parameter-dependent full-order controller $\Psi(\alpha)$ is irrelevant in the analysis conditions (9) and (10), the synthesis conditions presented in the next subsection require a *stabilizing* controller $\Psi(\alpha)$, which is clarified in the discussion below Theorem 4 in the next subsection.

B. Robust $\mathscr{H}_2$ and $\mathscr{H}_\infty$ synthesis

This subsection presents parameter-dependent LMI conditions to design robust controllers (2) for the uncertain linear system (1), such that an upper bound on the worst-case closed-loop $\mathscr{H}_2$ or $\mathscr{H}_\infty$ performance is guaranteed.

Theorem 3 (Robust $\mathscr{H}_2$ synthesis conditions): Let $\Psi(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_y)}$ correspond to a stabilizing parameter-dependent full-order controller for system (1), and let $\mathscr{A}_\Psi(\alpha)$, $\mathscr{B}_\Psi(\alpha)$, $\mathscr{C}_\Psi(\alpha)$ and $\mathscr{D}_\Psi(\alpha)$ denote the corresponding closed-loop matrices, as in (4). For a predefined controller order q $(0 \le q \le n_x)$, let $A_{22}(\alpha) \in \mathbb{R}^{(n_x-q)\times(n_x-q)}$ be a given Schur stable matrix for all $\alpha \in \Omega$. If there exist parameter-dependent matrices $P(\alpha) \in \mathbb{S}_+^{2n_x}$, $W(\alpha) \in \mathbb{S}^{n_z}$,

$$\bar{\Theta}(\alpha) = \begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{12}(\alpha) & \bar{\Theta}_{13} \\ 0 & 0_{(n_x-q)\times(n_x-q)} & 0 \\ \bar{\Theta}_{21} & \bar{\Theta}_{22}(\alpha) & \bar{\Theta}_{23} \end{bmatrix} \tag{11}$$

with $\bar{\Theta}_{11} \in \mathbb{R}^{q\times q}$, $\bar{\Theta}_{12}(\alpha) \in \mathbb{R}^{q\times(n_x-q)}$ and $\bar{\Theta}_{23} \in \mathbb{R}^{n_u\times n_y}$, and

$$Y(\alpha) = \begin{bmatrix} Y_{11} & Y_{12}(\alpha) & Y_{13} \\ 0 & Y_{22}(\alpha) & 0 \\ Y_{31} & Y_{32}(\alpha) & Y_{33} \end{bmatrix} \tag{12}$$

with $Y_{11} \in \mathbb{R}^{q\times q}$, $Y_{22}(\alpha) \in \mathbb{R}^{(n_x-q)\times(n_x-q)}$, and $Y_{33} \in \mathbb{R}^{n_u\times n_u}$, for all $\alpha \in \Omega$, and a scalar μ such that $\mathrm{Trace}\{W(\alpha)\} < \mu$ and the parameter-dependent LMIs (13) on page 4 hold for all $\alpha \in \Omega$, where $Z(\alpha)$ is given by

$$Z(\alpha) := \bar{\Theta}(\alpha) + Y(\alpha)\left(\begin{bmatrix} 0_{q\times q} & 0 & 0 \\ 0 & A_{22}(\alpha) & 0 \\ 0 & 0 & 0_{n_u\times n_y} \end{bmatrix} - \Psi(\alpha) \right), \tag{14}$$

then the robust controller parameterized by

$$\Theta = \begin{bmatrix} Y_{11} & Y_{13} \\ Y_{31} & Y_{33} \end{bmatrix}^{-1} \begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{13} \\ \bar{\Theta}_{21} & \bar{\Theta}_{23} \end{bmatrix} \tag{15}$$

stabilizes the closed-loop system (4) with a guaranteed upper bound $\sqrt{\mu}$ on its $\mathscr{H}_2$ performance for all $\alpha \in \Omega$.

Theorem 4 (Robust $\mathscr{H}_\infty$ synthesis conditions): Let $\Psi(\alpha) \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_y)}$ correspond to a stabilizing parameter-dependent full-order controller for system (1), and let $\mathscr{A}_\Psi(\alpha)$, $\mathscr{B}_\Psi(\alpha)$, $\mathscr{C}_\Psi(\alpha)$ and $\mathscr{D}_\Psi(\alpha)$ denote the corresponding closed-loop matrices, as in (4). For a predefined controller order q $(0 \le q \le n_x)$, let $A_{22}(\alpha) \in \mathbb{R}^{(n_x-q)\times(n_x-q)}$ be given Schur stable matrix for all $\alpha \in \Omega$. If there exist parameter-dependent matrices $P(\alpha) \in \mathbb{S}_+^{2n_x}$, $\bar{\Theta}(\alpha) \in \mathbb{R}^{(q+n_u)\times(n_x+n_y)}$, as in (11), with $\bar{\Theta}_{11} \in \mathbb{R}^{q\times q}$, $\bar{\Theta}_{12}(\alpha) \in \mathbb{R}^{q\times(n_x-q)}$ and $\bar{\Theta}_{23} \in \mathbb{R}^{n_u\times n_y}$, and $Y(\alpha)$, as in (12), with $Y_{11} \in \mathbb{R}^{q\times q}$, $Y_{22}(\alpha) \in \mathbb{R}^{(n_x-q)\times(n_x-q)}$, and $Y_{33} \in \mathbb{R}^{n_u\times n_u}$, for all $\alpha \in \Omega$, and a scalar γ such that the parameter-dependent LMI (16) on page 4 holds for all $\alpha \in \Omega$, where $Z(\alpha)$ is given by (14), then the robust controller parameterized by (15) stabilizes the closed-loop system (4) with a guaranteed upper bound $\sqrt{\gamma}$ on its $\mathscr{H}_\infty$ performance for all $\alpha \in \Omega$.

We omit the proofs of Theorem 3 and Theorem 4, since they are straightforward extensions of the proofs presented in [5] to the case of uncertain systems. The synthesis conditions (13) and (16) feature additional conservatism compared to the analysis conditions (9) and (10), respectively. Namely, a nonlinear change of variables is necessary and structural constraints need to be imposed on the slack variables $X_j(\alpha)$ in (9) and (10) to render the synthesis conditions convex. Specifically, the derivation of the synthesis conditions (13) relies on the selections $X_j(\alpha) = 0$ for $j = 1,2,4,5,6$, $X_3(\alpha) = Y(\alpha)$ and $X_7(\alpha) = -Y(\alpha)$ in the analysis conditions (9). Similarly, $X_j(\alpha) = 0$ for $j = 1,2$ and $X_3(\alpha) = Y(\alpha)$ are selected in the analysis condition (10) to arrive at (16). To

$$Q_\infty(\alpha,\Psi(\alpha))' \begin{bmatrix} -P(\alpha) & 0 & 0 & 0 \\ 0 & P(\alpha) & 0 & 0 \\ 0 & 0 & -\gamma I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} Q_\infty(\alpha,\Psi(\alpha)) + \mathrm{He}\left\{ \begin{bmatrix} X_1(\alpha) \\ X_2(\alpha) \\ X_3(\alpha) \end{bmatrix} \begin{bmatrix} \Upsilon(\alpha)\tilde{C}_y(\alpha) & \Upsilon(\alpha)\tilde{D}_y(\alpha) & -I \end{bmatrix} \right\} \prec 0 \tag{10}$$

$$Q_2(\alpha,\Psi(\alpha))' \begin{bmatrix} -P(\alpha) & 0 & 0 \\ 0 & P(\alpha) & 0 \\ 0 & 0 & -I \end{bmatrix} Q_2(\alpha,\Psi(\alpha)) + \mathrm{He}\left\{ \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \begin{bmatrix} Z(\alpha)\tilde{C}_y(\alpha) & Z(\alpha)\tilde{D}_y(\alpha) & -Y(\alpha) \end{bmatrix} \right\} \prec 0 \tag{13a}$$

$$\begin{bmatrix} W(\alpha) & \mathscr{C}_\Psi(\alpha) & \mathscr{D}_\Psi(\alpha) & \tilde{D}_u(\alpha) \\ \star & P(\alpha) & 0 & 0 \\ \star & \star & I & 0 \\ \star & \star & \star & 0 \end{bmatrix} + \mathrm{He}\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ I \end{bmatrix} \begin{bmatrix} 0 & -Z(\alpha)\tilde{C}_y(\alpha) & -Z(\alpha)\tilde{D}_y(\alpha) & Y(\alpha) \end{bmatrix} \right\} \succ 0 \tag{13b}$$

$$Q_\infty(\alpha,\Psi(\alpha))' \begin{bmatrix} -P(\alpha) & 0 & 0 & 0 \\ 0 & P(\alpha) & 0 & 0 \\ 0 & 0 & -\gamma I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} Q_\infty(\alpha,\Psi(\alpha)) + \mathrm{He}\left\{ \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \begin{bmatrix} Z(\alpha)\tilde{C}_y(\alpha) & Z(\alpha)\tilde{D}_y(\alpha) & -Y(\alpha) \end{bmatrix} \right\} \prec 0 \tag{16}$$

allow the reconstruction of a robust controller Θ through the nonlinear transformation (15), note that the specific structures (12) on $Y(\alpha)$ and (7) on $\Theta_a(\alpha)$ (which imply the structure (11) on $\bar{\Theta}(\alpha)$) are imposed.

Note that the parameter-dependent LMIs (13) (LMI (16)) are feasible for all $\alpha \in \Omega$ only if the closed-loop system $H_\Psi(\alpha)$ is exponentially stable and satisfies the performance bound $\|H_\Psi(\alpha)\|_2^2 < \mu$ ($\|H_\Psi(\alpha)\|_\infty^2 < \gamma$) for all $\alpha \in \Omega$, as can be seen from application of the projection lemma [15]. Therefore, these LMIs require a stabilizing parameter-dependent controller $\Psi(\alpha)$ for the computation of a robust fixed-order controller Θ.

The parameter-dependent LMI conditions (13) and (16) are *semi-infinite*, i.e., they should hold for infinitely many parameter values $\alpha \in \Omega$, resulting in an infinite number of constraints. A finite set of sufficient LMIs that guarantee feasibility of the parameter-dependent LMIs (13) and (16) is derived by exploiting the polytopic structure of Ω. Namely, we express each point $\alpha \in \Omega$ as a convex combination of the vertices of Ω, and impose positivity/negativity on the coefficients of the resulting homogenized polynomially parameter-dependent LMIs [17]. This finite set of LMI constraints can be automatically determined by using, e.g., the Robust LMI Parser [18].

C. Multi-objective robust control

Incorporation of multiple (usually conflicting) control design objectives is indispensable for many practical applications. Therefore, this subsection explains how the synthesis conditions (13) and (16) are adapted to handle multi-objective robust $\mathscr{H}_2/\mathscr{H}_\infty$ control problems. Such problems include, for instance, the minimization of a worst-case $\mathscr{H}_\infty$ performance bound subject to an a priori defined worst-case $\mathscr{H}_2$ performance bound.

We define each $\mathscr{H}_2$ ($\mathscr{H}_\infty$) performance specification by an index $j \in \mathscr{J}_{\mathscr{H}_2}$ ($j \in \mathscr{J}_{\mathscr{H}_\infty}$), and define the set containing the indices of all performances by $\mathscr{J} = \mathscr{J}_{\mathscr{H}_2} \cup \mathscr{J}_{\mathscr{H}_\infty}$. Each performance specification $j \in \mathscr{J}$ is imposed by appropriately defining selection matrices L_j and R_j and selecting an input-output channel $w_j \to z_j$ of the uncertain linear system (1) as follows

$$\begin{cases} x(k+1) = A(\alpha)x(k) + B_w(\alpha)w_j(k) + B_u(\alpha)u(k), \\ \quad z_j(k) = L_j C_z(\alpha)x(k) + L_j D_w(\alpha)w_j(k) + L_j D_u(\alpha)u(k), \\ \quad y(k) = C_y(\alpha)x(k) + D_y(\alpha)w_j(k), \end{cases} \tag{17}$$

where $w_j(k) := R_j w(k)$ and $z_j(k) := L_j z(k)$. Subsequently, the synthesis conditions (13) (for $j \in \mathscr{J}_{\mathscr{H}_2}$) and (16) (for $j \in \mathscr{J}_{\mathscr{H}_\infty}$) are imposed for each of the uncertain systems (17), $j \in \mathscr{J}$.

Since the reconstructed multi-objective controller depends on the optimization variables Y_{11}, Y_{13}, Y_{31}, Y_{33}, $\bar{\Theta}_{11}$, $\bar{\Theta}_{13}$, $\bar{\Theta}_{21}$ and $\bar{\Theta}_{23}$ (see (15)), these are chosen identical for all $j \in \mathscr{J}$, introducing additional conservatism with respect to single-objective synthesis. However, the remaining optimization variables are chosen differently for each performance specification (i.e., we define the variables $P_j(\alpha)$, $\bar{\Theta}_{12,j}$, $\bar{\Theta}_{22,j}$, $Y_{12,j}(\alpha)$, $Y_{22,j}(\alpha)$, $Y_{32,j}(\alpha)$ for $j \in \mathscr{J}$, μ_j, $W_j(\alpha)$ for $j \in \mathscr{J}_{\mathscr{H}_2}$, and γ_j for $j \in \mathscr{J}_{\mathscr{H}_\infty}$), since convexity is then retained while keeping conservatism to a minimum. Note that it is also possible to use different initial controllers $\Psi_j(\alpha)$ and matrices $A_{22,j}(\alpha)$ for the different performance channels.

D. Iterative LMI procedure

A convex procedure to iteratively reduce conservatism in a fixed-order robust $\mathscr{H}_2/\mathscr{H}_\infty$ control design is presented now. For the sake of clarity, we consider the single-objective fixed-order robust $\mathscr{H}_\infty$ control problem. The extension to handle $\mathscr{H}_2$ or multi-objective controller designs is straightforward.

Starting from an a priori computed stabilizing full-order parameter-dependent controller $\Psi(\alpha)$ for the uncertain system (1) (e.g., see [13], [14]), and a given Schur stable matrix $A_{22}(\alpha)$, suppose that the synthesis condition (16) provides a feasible solution. Then, by alternately solving the analysis condition (10) in the optimization variables $P(\alpha)$, $X_1(\alpha)$, $X_2(\alpha)$, $X_3(\alpha)$, respectively, $P(\alpha)$, $\Theta_a(\alpha)$. The matrix $\Psi(\alpha)$

is fixed in the analysis LMI (10), since it does not influence the solution.

Specifically, the following LMI procedure is applied to iteratively compute robust controllers of the same order guaranteeing a better worst-case $\mathscr{H}_\infty$ performance:

1) Using the convex synthesis condition (16), compute a robust controller $\Theta^{(0)}$ with a preselected fixed order q and a guaranteed worst-case $\mathscr{H}_\infty$ performance $\gamma^{(0)}$.
2) Set $k := 1$.
3) Substitute the solution variable $\Theta^{(k-1)}$ in the analysis condition (10), and optimize the performance bound $\gamma_{\mathrm{a}}^{(k)}$ over the Lyapunov matrix $P(\alpha)$ and the slack variables $X_j(\alpha)$, $j = 1,2,3$. Since substitution of the solution corresponding to the previous step implies that the constraint (10) is satisfied for $\gamma_{\mathrm{a}}^{(k)} = \gamma^{(k-1)}$, we obtain $\gamma_{\mathrm{a}}^{(k)} \leq \gamma^{(k-1)}$.
4) Substitute the solution variables $X_j(\alpha)$ (of the previous step) in the analysis condition (10), and optimize the performance bound $\gamma^{(k)}$ over the Lyapunov matrix $P(\alpha)$ and the controller variables $\Theta^{(k)}$, $A_{12}(\alpha)$, $A_{22}(\alpha)$ and $C_2(\alpha)$ (see (7)). Since $\Theta^{(k-1)}$ is a solution, we get that $\gamma^{(k)} \leq \gamma_{\mathrm{a}}^{(k)}$.
5) If $|\gamma^{(k)} - \gamma^{(k-1)}|/\gamma^{(k-1)} < \varepsilon$, with ε a predefined tolerance, stop. Else, set $k := k+1$ and return to step 3.

IV. NUMERICAL VALIDATION

This section considers some numerical examples to validate the fixed-order robust $\mathscr{H}_2/\mathscr{H}_\infty$ controller design approach presented in Section III, by means of comparisons with existing approaches. The LMIs are implemented and solved in MATLAB using the software packages Yalmip [19] and SeDuMi [20].

A. Example I

Consider the discretized mass-spring-damper system from Example II in [21], consisting of two masses $m_1 = 2$ [kg] and $m_2 = 1$ [kg], two springs with coefficients $k_1 \in [1,4]$ [N/m] and $k_2 = 0.5$ [N/m], and a damper with uncertain damping constant $d \in [1,4]$ [Ns/m]. The dynamics are expressed in the form (1) as follows

$$A(\alpha) = \begin{bmatrix} 1 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -\frac{0.1(k_1+k_2)}{m_1} & \frac{0.1k_2}{m_1} & 1-\frac{0.1d}{m_1} & 0 \\ \frac{0.1k_2}{m_2} & -\frac{0.1k_2}{m_2} & 0 & 1-\frac{0.1d}{m_2} \end{bmatrix}$$

$$B_w = \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \\ 0 \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ 0 \\ \frac{0.1}{m_1} \\ 0 \end{bmatrix}, C_z = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}', C_y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}',$$

$D_{zw} = 0$, $D_{zu} = 0$ and $D_{yw} = 0$, where only the A-matrix is uncertain and affinely dependends on the two-dimensional parameter $\alpha := \begin{bmatrix} k_1 & d \end{bmatrix}' \in [1,4] \times [1,4] = \Omega$.

The aim is to compute fixed-order robust controllers with optimal closed-loop $\mathscr{H}_\infty$ performance. As a starting point, a stabilizing full-order controller $\Psi(\alpha)$ with an affine dependency on α is computed with the approach [13]. Minimization of the $\mathscr{H}_\infty$ bound yields a parameter-dependent controller $\Psi(\alpha)$ for which the LMI (16) is infeasible. Therefore, we compute a suboptimal parameter-dependent $\mathscr{H}_\infty$ controller by fixing the $\mathscr{H}_\infty$ performance bound $\gamma = 12$ in the associated $\mathscr{H}_\infty$ synthesis LMIs and solving the corresponding feasibility problem, resulting in a suboptimal parameter-dependent controller with a guaranteed $\mathscr{H}_\infty$ performance of 7.66. Subsequently, fixed-order robust controllers of all orders $q = 0,\ldots,4$ are computed by substituting $\Psi(\alpha)$ and $A_{22}(\alpha) = 0$ in the synthesis condition (16), and selecting an affine parameterization for the parameter-dependent LMI variables. The resulting worst-case $\mathscr{H}_\infty$ bounds are shown in the second row of Table I, and are subject to conservatism (especially for $q = 0$). Therefore, the corresponding fixed-order robust controllers are used as a starting point in the iterative procedure proposed in Subsection III-D to compute less conservative robust controllers, taking all pameter-dependent LMI variables affine in α, and defining a tolerance $\varepsilon = 10^{-3}$. The third row of Table I shows the $\mathscr{H}_\infty$ bound $\gamma^{(k)}$ that is obtained after k iterations, and reveals a significant reduction of conservatism for all orders. Compared to the robust static output feedback design approach [21], which provided a $\mathscr{H}_\infty$ performance bound of 8.54 (i.e., for the case $q = 0$), we achieved a relative improvement of 12%. Moreover, no feasible solution was obtained with the robust static output feedback design approaches [22], [23], [24].

TABLE I

WORST-CASE $\mathscr{H}_\infty$ PERFORMANCE BOUNDS FOR EACH ORDER $q = 0,\ldots,4$, RESULTING FROM THEOREM 4 ($\gamma^{(0)}$) AND SUBSEQUENT APPLICATION OF THE ITERATIVE PROCEDURE ($\gamma^{(k)}$, WITH k THE NUMBER OF ITERATIONS).

q	4	3	2	1	0
$\gamma^{(0)}$	9.39	15.5	10.3	14.0	4.39×10^4
$\gamma^{(k)}$ (k)	6.60 (15)	6.60 (40)	6.85 (34)	7.55 (8)	7.55 (20)

B. Example II

We consider a slightly modified version of the 3rd order LTI model used in Example 4 of [22]:

$$x(k+1) = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0.5 & 0 \\ 0 & 1 & -\alpha_1 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} w_2(k) + \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}' w_\infty(k) + \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}' u(k),$$

with exogenous outputs $z_2(k) = \begin{bmatrix} x(k)' & u(k) \end{bmatrix}'$, $z_\infty(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(k)$, and measurement equation

$$y(k) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0 & \alpha_2 \end{bmatrix} w_2(k),$$

where $\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix}' \in [0.45, 0.55] \times [0.9, 1.1] = \Omega$.

The goal is to compute a robust full-order controller minimizing a bound μ on the worst-case $\mathscr{H}_2$ performance from w_2 to z_2, while an a priori imposed bound $\gamma = 3.5$ on the worst-case $\mathscr{H}_\infty$ performance from w_∞ to z_∞ is satisfied. Note

TABLE II

COMPARISON OF THE WORST-CASE $\mathscr{H}_2$ PERFORMANCE BOUNDS RESULTING FROM SYNTHESIS (μ) AND A POSTERIORI ANALYSIS (μ_{ANA}), AND THE PREFIXED (A POSTERIORI COMPUTED) $\mathscr{H}_\infty$ BOUNDS $\gamma\,(\gamma_{\text{ANA}})$, CORRESPONDING TO THE PARAMETER-DEPENDENT, ROBUST, AND IMPROVED ROBUST CONTROLLER.

	μ	μ_{ana}	γ	γ_{ana}
Parameter Dependent [13]	19.04	16.35	3.5	3.08
Robust (Theorems 3 and 4)	17.90	16.38	3.5	3.37
Improved Robust (10 iterations)	16.33	16.31	3.5	3.35

that, due to an uncertain matrix relating w_2 to y, the approach [10] cannot be applied. First, a multi-objective parameter-dependent controller with an affine parameter-dependency is computed with the approach [13], resulting in the $\mathscr{H}_2$ bound $\mu = 19.04$, as shown in Table II. A tighter bound $\mu_{\text{ana}} = 16.35$ is computed by a posteriori solving an analysis LMI. Substituting the parameter-dependent controller for $\Psi(\alpha)$ (and $A_{22}(\alpha) = 0$) in the synthesis conditions of Theorem 3 and 4, a full-order robust controller guaranteeing a closed-loop $\mathscr{H}_\infty$ performance of 17.90 is computed, and a corresponding tighter bound $\mu_{\text{ana}} = 16.38$. Applying the iterative procedure with $\varepsilon = 10^{-4}$, a robust controller with a $\mathscr{H}_\infty$ bound $\mu = 16.33$ and $\mu_{\text{ana}} = 16.31$ is computed in 10 iterations, outperforming the parameter-dependent controller.

V. CONCLUSIONS

An LMI framework to design robust $\mathscr{H}_2/\mathscr{H}_\infty$ controllers for discrete-time LTI systems with parametric uncertainty is presented. Starting from an a priori computed parameter-dependent full-order controller stabilizing the uncertain system for all parameter values, sufficient LMIs for fixed-order robust $\mathscr{H}_2/\mathscr{H}_\infty$ analysis and synthesis are derived. Furthermore, a procedure using these LMIs is proposed to iteratively reduce conservatism in a fixed-order robust control design. The potential of the robust controller design procedure is illustrated by numerical comparisons with existing approaches.

REFERENCES

[1] Q. Tran Dinh, S. Gumussoy, W. Michiels, and M. Diehl, "Combining convex-concave decompositions and linearization approaches for solving BMIs, with application to static output feedback," *IEEE Transactions on Automatic Control*, vol. 57, no. 6, pp. 1377–1390, 2011.

[2] P. Apkarian and D. Noll, "Nonsmooth $\mathscr{H}_\infty$ synthesis," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 71–86, January 2006.

[3] S. Gumussoy, D. Henrion, M. Millstone, and M. L. Overton, "Multi-objective robust control with HIFOO 2.0," in *Proceedings of the 6th IFAC Symposium on Robust Control Design*, Haifa, Israel, June 2009.

[4] D. Peaucelle and D. Arzelier, "An efficient numerical solution for $\mathscr{H}_2$ static output feedback synthesis," in *Proceedings of the 2001 European Control Conference*, Porto, Portugal, September 2001.

[5] G. Hilhorst, G. Pipeleers, W. Michiels, and J. Swevers, "Sufficient LMI conditions for reduced-order multi-objective $\mathscr{H}_2/\mathscr{H}_\infty$ control of LTI systems," *European Journal of Control*, vol. 23, pp. 17–25, May 2015.

[6] D. Peaucelle, D. Arzelier, O. Bachelier, and J. Bernussou, "A new robust D-stability condition for real convex polytopic uncertainty," *Systems & Control Letters*, vol. 40, no. 1, pp. 21–30, 2000.

[7] C. M. Agulhari, R. C. L. F. Oliveira, and P. L. D. Peres, "LMI relaxations for reduced-order robust $\mathscr{H}_\infty$ control of continuous-time uncertain linear systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 6, pp. 1532–1537, 2012.

[8] U. Shaked, "A LPV approach to robust $\mathscr{H}_2$ and $\mathscr{H}_\infty$ static output-feedback design," in *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada, USA, December 2002, pp. 3476–3481.

[9] H. Köroğlu and P. Falcone, "New LMI conditions for static output feedback synthesis with multiple performance objectives," in *Proceedings of the 53rd IEEE Conference on Decision and Control*, December 2014, pp. 866–871.

[10] M. S. Sadabadi and A. Karimi, "An LMI formulation of fixed-order $\mathscr{H}_\infty$ and $\mathscr{H}_2$ controller design for discrete-time systems with polytopic uncertainty," in *Proceedings of the 52nd IEEE Conference on Decision and Control*, Florence, Italy, December 2013, pp. 2453–2458.

[11] G. Hilhorst, G. Pipeleers, W. Michiels, R. C. L. F. Oliveira, P. L. D. Peres, and J. Swevers, "Reduced-order $\mathscr{H}_2/\mathscr{H}_\infty$ control of discrete-time LPV systems with experimental validation on an overhead crane test setup," in *Proceedings of the 2015 American Control Conference*, Chicago, IL, USA, July 2015, pp. 125–130.

[12] G. Hilhorst, G. Pipeleers, W. Michiels, R. Oliveira, P. Peres, and J. Swevers, "Structured linear parameter-varying feedback control of a lab-scale overhead crane," *IEEE Transactions on Control Systems Technology*, (under revision).

[13] J. De Caigny, J. F. Camino, R. C. L. F. Oliveira, P. L. D. Peres, and J. Swevers, "Gain-scheduled dynamic output feedback for discrete-time LPV systems," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 5, pp. 535–558, 2011.

[14] P. Apkarian, P. C. Pellanda, and H. D. Tuan, "Mixed $\mathscr{H}_2/\mathscr{H}_\infty$ multi-channel linear parameter-varying control in discrete time," *Systems & Control Letters*, vol. 41, no. 5, pp. 333–346, 2000.

[15] P. Apkarian and P. Gahinet, "A convex characterization of gain-scheduled $\mathscr{H}_\infty$ controllers," *IEEE Transactions on Automatic Control*, vol. 40, no. 5, pp. 853–864, 1995.

[16] E. Feron, P. Apkarian, and P. Gahinet, "Analysis and synthesis of robust control systems via parameter-dependent Lyapunov functions," *IEEE Transactions on Automatic Control*, vol. 41, no. 7, pp. 1041–1046, July 1996.

[17] R. C. L. F. Oliveira and P. L. D. Peres, "Parameter-dependent LMIs in robust analysis: Characterization of homogeneous polynomially parameter-dependent solutions via LMI relaxations," *IEEE Transactions on Automatic Control*, vol. 52, pp. 1334–1340, 2007.

[18] C. M. Agulhari, R. C. L. F. Oliveira, and P. L. D. Peres, "Robust LMI parser: A computational package to construct LMI conditions for uncertain systems," in *Proceedings of the XIX Brazilian Conference on Automation (CBA 2012)*, Campina Grande, PB, Brasil, September 2012.

[19] J. Löfberg, "Yalmip : A toolbox for modeling and optimization in MATLAB," in *Proceedings of the CACSD Conference*, Taipei, Taiwan, 2004, pp. 284–289.

[20] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optimization Methods and Software*, vol. 11–12, pp. 625–653, 1999.

[21] C. M. Agulhari, R. C. L. F. Oliveira, and P. L. D. Peres, "Robust $\mathscr{H}_\infty$ static output-feedback design for time-invariant discrete-time polytopic systems from parameter-dependent state-feedback gains," in *Proceedings of the 2010 American Control Conference*, Baltimore, MD, USA, July 2010, pp. 4677–4682.

[22] M. C. de Oliveira, J. C. Geromel, and J. Bernussou, "Extended $\mathscr{H}_2$ and $\mathscr{H}_\infty$ norm characterizations and controller parametrizations for discrete-time systems," *International Journal of Control*, vol. 75, no. 9, pp. 666–679, 2002.

[23] J. Dong and G.-H. Yang, "Robust static output feedback control for linear discrete-time systems with time-varying uncertainties," *Systems & Control Letters*, vol. 57, pp. 123–131, 2008.

[24] X. Du and G.-H. Yang, "LMI conditions for $\mathscr{H}_\infty$ static output feedback control of discrete-time systems," in *Proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, December 2008, pp. 5450–5455.